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Higher order computational model for the thermo-elastic analysis of cross-ply laminated composite plates

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Abstract— Analytical formulations and solutions for the stress analysis of simply supported cross-ply laminated composite plates subjected to thermal load based on higher order refined theory are presented. In addition, another higher order theory and the first-order theory developed by other investigators and already available in the literature are also considered for the evaluation. The equation of equilibrium is obtained using Principal of Minimum Potential Energy (PMPE). Solutions are obtained in closed form using Navier's technique by solving the boundary value problem. The transverse stresses are obtained by integrating equilibrium equations. Plates with different aspect ratio are studied. Numerical results are presented for the displacments and the stresses.

Index Terms— Analytical solution, Composite plates, Higher-order theory, Navier's solution, Stress analysis, Thermo-elastic, Thermal load .

1 Introduction

MULTI-LAYERED plates made up of composite materials are widely used in aerospace, aeronautical, automobiles and other hi-tech industries. Mathematical modeling and behavior of these structural components subjected to severe thermal loading has attracted considerable attention. Delamination of layers and longitudinal cracks in the matrix are predominant cause of failure of composite plates subjected to severe thermal loading, therefore developing very accurate and efficient theoretical model for thermo-elastic analysis of composite plates have constantly been an important area of research. The thermal-membrane coupling effect was found to be very significant in the thermo-elastic analysis of antisymmetric cross ply and angle ply laminates [1]. The finite element formulations and solutions using first order shear deformation theory (FSDT) and penalty finite element was presented for the thermal analysis of multi-layered plates [2]. A generalized Levy type solution in combination with state-space method is used to analyses the thermal bending of cross-ply laminated plates [3]. To get complete insight in to this area, researchers may refer to the review article on the various computational models used for the thermo-elastic analysis of multi-layered plates [4], [5]. A discrete-layer shear deformation laminated plate theory is used to analyses steady-state thermal stresses in laminated plates [6]. A displacement centered higher order theory which employs realistic displacement variations through the thickness is presented in [7]. In-order to overcome the limitation of classical and first order shear deformation theory, globa-higher-order based on power series for the evaluation of inter-laminar stresses subjected to thermal loading have been devel oped by [8]. A global-local higher order theory combined with finite element method is been used to capture the response details of laminate subjected to thermal loading [9]. In this paper, an attempt has been made to compare and assess quantitatively the accuracy of the results obtained using the various higher order models in predicating the thermal stresses of simply supported cross-ply laminated composite plates subjected to thermal loading.

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2 Displacement models

The following two higher order and the first order shear deformation models are considered.

PRESENT [10]

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\,\theta_x(x,y) + z^2 u_0^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_0(x,y) + z\,\theta_y(x,y) + z^2 v_0^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (1)$$

HSDT5 [11]

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\left[\theta_x(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left\{\theta_x(x,y) + \frac{\partial w_0}{\partial x}\right\}\right] \\ v(x,y,z) &= v_0(x,y) + z\left[\theta_y(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left\{\theta_y(x,y) + \frac{\partial w_0}{\partial y}\right\}\right] \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (2)$$

FSDT [12]

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\,\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\,\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (3)$$

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The parameters u_0, v_0 are the in-plane displacements and w_0 is the transverse displacement of a point (x, y) on the middle plane (z=0). The functions θ_x, θ_y are the rotations of the normal to the middle plane about y- and x- axes, respectively. The parameters u_0^*, v_0^*, θ_x^*, θ_y^* are the higher-order terms in the Taylor's series expansion and they represent higher-order transverse cross-sectional deformation modes.
The stress-strain relationship accounting for the transverse shear deformation and thermal effects is given by

$$\{\sigma\} = [Q]\{\varepsilon\} - [Q]\{\alpha\}\,\Delta T \quad (4)$$

Where, $\{\sigma\}$ =Stress vector

$[Q]$ =Transformed elastic coefficients

$\{\varepsilon\}$ =Strain vector

$\{\alpha\}$ =Thermal expansion coefficient vector

ΔT = Temperature rise in the laminate

The equations of equilibrium are obtained using principal of Minimum Potential Energy (PMPE). Solutions are obtained in closed form using Navier's technique by solving the boundary value problem. The in-plane stresses are computed using the constitutive relationship and the transverse stresses are obtained by integrating the 3D elasticity equilibrium equations.

3 NUMERICAL RESULTS AND DISCUSSION

In this section, the numerical example solved is described and discussed. A steady state thermo-elastic bending of a simply supported cross-ply laminated plate is considered for analysis.
The material properties and the thickness of each layer are uniform. The material constants considered are as follows [7]:

$$E_1/E_2 = 25, \quad E_2 = E_3 = 1, \quad G_{12} = G_{13} = 0.5, \quad G_{23} = 0.2$$

$$\nu_{12} = \nu_{13} = \nu_{23} = 0.25, \qquad \alpha_2/\alpha_1 = 1125$$

Results reported are using the following non-dimensional form:

$$\bar{w} = \frac{w}{h\alpha_1 T_0 S^2} \qquad \bar{u} = \frac{u}{h\alpha_1 T_0 S} \qquad \bar{v} = \frac{v}{h\alpha_1 T_0 S}$$

$$\bar{\sigma}_x = \frac{\sigma_x}{E_2\alpha_1 T_0} \qquad \bar{\sigma}_y = \frac{\sigma_y}{E_2\alpha_1 T_0} \qquad \bar{\tau}_{xy} = \frac{\tau_{xy}}{E_2\alpha_1 T_0}$$

$$\bar{\tau}_{xz} = \frac{\tau_{xz}}{E_2\alpha_1 T_0} \qquad \bar{\tau}_{yz} = \frac{\tau_{yz}}{E_2\alpha_1 T_0} \qquad \text{Where,} \quad S = \frac{a}{h}$$

Unless otherwise specified within the table the location (i.e. x,y and z coordinates) for values of displacements and stresses for present evaluation are as follows:

In-plane displacements (u) : $(0, b/2, -h/2)$
In-plane displacements (v) : $(a/2, 0, -h/2)$
Transverse displacement (w) : $(a/2, b/2, \pm h/2)$
In-plane normal stress (σ_x) : $(a/2, b/2, +h/2)$
In-plane normal stress (σ_y) : $(a/2, b/2, -h/2)$
In-plane shear stress (τ_{xy}) : $(0, 0, -h/2)$
Transverse shear stress (τ_{xz}) : $(0, b/2, \pm h/6)$
Transverse shear stress (τ_{yz}) : $(a/2, 0, \pm h/6)$

Example: A steady-state thermo-elastic bending of a simply supported three layer cross-ply (0/90/0) square laminated plate (a=b) is analyzed.

TABLE 1
IN-PLANE AND TRANSVERSE DISPLACEMENTS FOR THERMAL LOADING

a/h	MODEL	$\bar{u}$	$\bar{v}$	$\bar{w}$
4	PRESENT	14.58	71.88	25.67
	HSDT5	14.45	62.52	25.78
	FSDT	9.17	71.14	22.75
	EXACT [7]	18.11	81.83	42.69
10	PRESENT	15.85	29.49	14.17
	HSDT5	15.58	27.87	14.11
	FSDT	14.30	29.70	13.30
	EXACT [7]	16.61	31.95	17.39
20	PRESENT	15.96	19.66	11.27
	HSDT5	15.88	19.24	11.25
	FSDT	15.53	19.71	11.03
	EXACT [7]	16.17	20.34	12.12
50	PRESENT	15.99	16.59	10.36
	HSDT5	15.97	16.52	10.36
	FSDT	15.92	16.60	10.32
	EXACT [7]	16.02	16.71	10.50

TABLE 2
IN-PLANE AND TRANSVERSE STRESSES FOR THERMAL LOADING

a/h	MODEL	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
4	PRESENT	898	890.2	135.8	94.5	-135.74
	HSDT5	880.1	919.8	120.9	95.6	-137.78
	FSDT	471.0	896.8	126.2	104.2	-134.50
	EXACT [7]	1183	856.1	157.0	84.81	-121.87
10	PRESENT	964	1023	71.23	62.16	-66.65
	HSDT5	942	1028	68.26	62.52	-66.82
	FSDT	842	1023	69.10	63.60	-66.61
	EXACT [7]	1026	1014	76.29	60.54	-66.01
20	PRESENT	965	1054	55.95	34.23	-34.86
	HSDT5	958	1055	55.16	34.29	-34.88
	FSDT	931	1054	55.36	34.44	-34.85
	EXACT [7]	982	1051	57.35	33.98	-34.76
50	PRESENT	964.9	1063	51.17	14.09	-14.13
	HSDT5	963.7	1063	51.05	14.10	-14.13

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	FSDT	959.3	1063	51.08	14.11	-14.13
	EXACT [7]	967.5	1063	51.41	14.07	-14.13

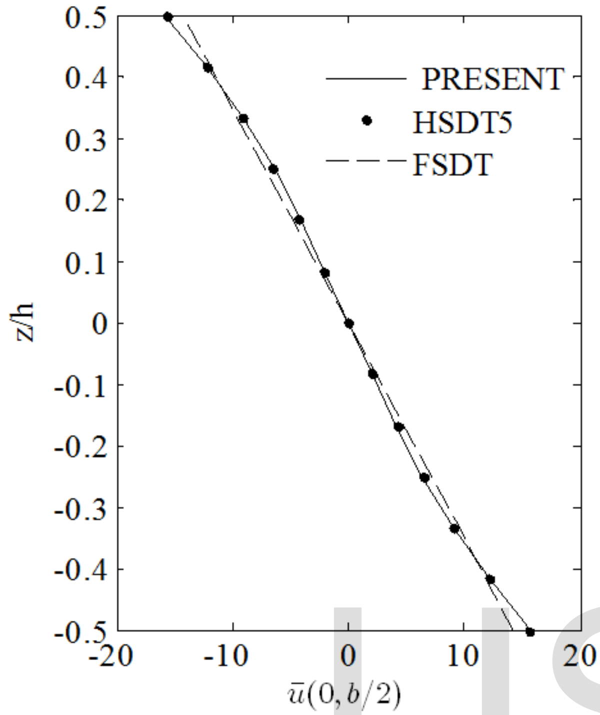

Fig. 1. Through thickness variation of In-plane displacement $\bar{u}$ for ratio of a/h=10

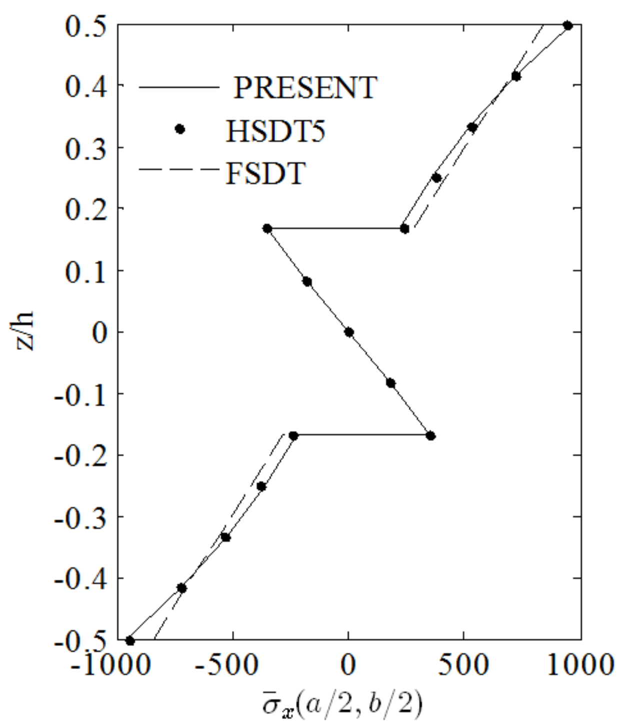

Fig. 2. Through thickness variation of In-plane normal $\bar{\sigma}_x$ stress

for ratio of a/h=10

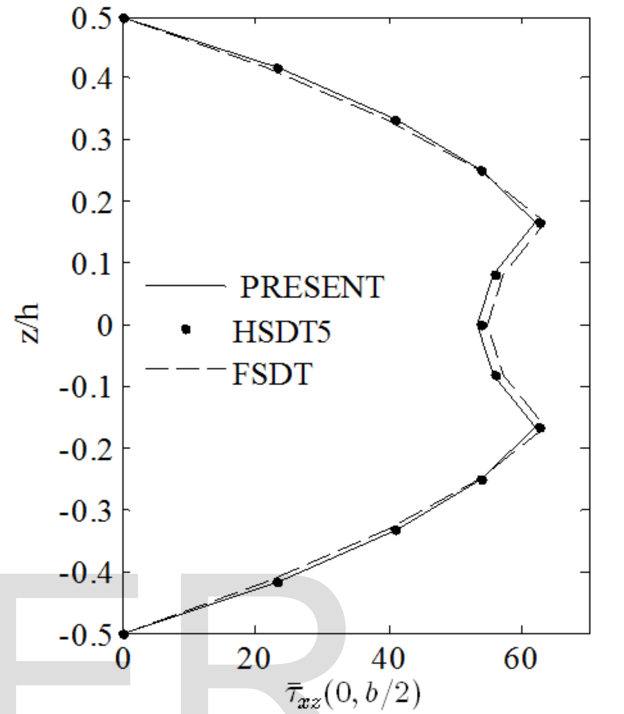

Fig. 3. Through thickness variation of Transverse shear stress $\bar{\tau}_{xz}$ for ratio of a/h=10

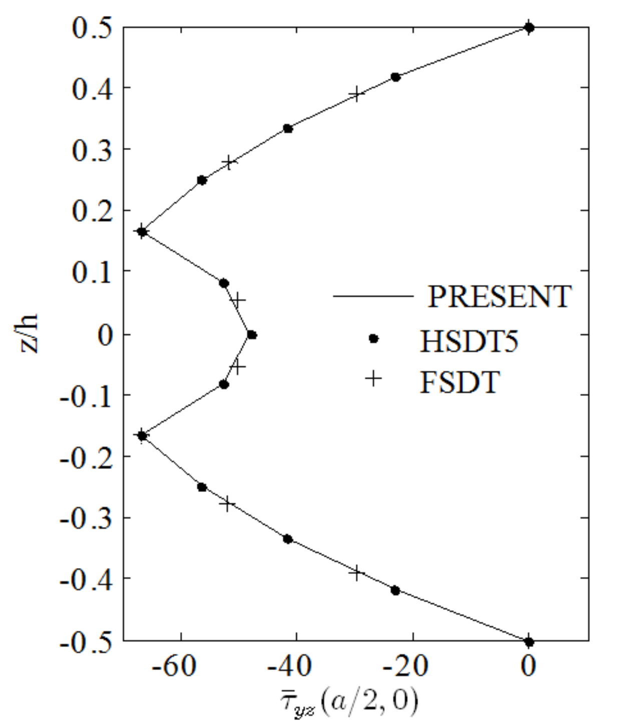

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Fig. 4. Through thickness variation of Transverse shear stress $\bar{\tau}_{yz}$ for ratio of a/h=10

The plates are loaded through a temperature distribution of the form:

$$\Delta T(x,y,z) = \frac{2T_0}{h} z \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

The non-dimensional values of in-plane and transverse displacements and the stresses for various values of a/h ratio are given in Table 1 and Table 2. It is found that the results generated using the three models are in good agreement with the exact three-dimensional elasticity solution for thick to relatively thin plates, whereas considerable difference in numerical values exists in the case of very thick plates (i.e. a/h=4). This is attributed to the fact that these models do not represent the higher-order transverse cross sectional deformation modes, which is very significant in thick plates. Fig. 1 represents through the thickness variation of in-plane displacement. The through the thickness variation of in-plane and transverse stresses are shown in Fig. 2, Fig. 3 and Fig. 4. It is found that the variation of all the three models are in close agreement with each other.

4 Conclusion

Analytical formulations and solutions for the thermal stress analysis of simply supported cross-ply laminated composite plates using higher- order shear deformation theory is presented. The maximum and through the thickness variation of displacements and stresses with varying side-to-thickness ratio are discussed. The accuracy of each model in prediciting the displacments and stresses are established by comparing the results with the three-dimensional elasticity solutions. The bench mark numerical results presented herein will provide a good reference for researchers working in the area of thermo-elastic analysis of composite plates.

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